NASA TT F-10,188

THERMAL CHARACTERISTICS OF A PEBBLE BED HEATER

NASA TT F-10,188

Nisiki Hayashi

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

National Aerospace Laboratory, Technical Report No. 52,
pp 1-15, Tokyo, 1963

FACILITY FORM 602

N66 27502 (ACCESSION NUMBER)

(THRU)

29 (PAGES)

1 (CODE)

(NASA CR OR TMX OR AD NUMBER)

33 (CATEGORY)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. JUNE 1966

NASA TT F-10,188

THERMAL CHARACTERISTICS OF A PEBBLE BED HEATER*

Nisiki Hayashi**

ABSTRACT

27502

The heat transfer between pebbles contained in a circular tube and the fluid moving through it are considered. Neglecting the effect of the heat capacity of the wall, several analyses are made for the following cases: (1) the initial pebble temperature is equal to that of the fluid and is uniform, (2) the thermal capacity per unit volume of fluid is far less than that of the pebbles, and the initial pebble temperature is a linear function of $e^{-c\S}$, where c is a constant and § is a nondimensional axial distance. Numerical calculations were performed for two pebble-bed heaters by means of an electronic digital computer.

Author

1. Introduction /1***

In supersonic wind tunnels, the air temperature of the measurement section becomes very low because of its adiabatic expansion in the accelera-

---

*Received September 25, 1963.

**University of Cincinnati, Department of Engineering, Aerospace Laboratory (Aerodynamics Section 1).

***Note: Numbers in the margin indicate pagination in the original foreign text.

ting range. Therefore, the air must be pre-heated to raise the stagnation temperature. Generally, the following three methods are used for this purpose:

(1) A method utilizing a metal electric heater, or a heat reservoir type heat exchanger;

(2) A method utilizing a non-metal heat reservoir type heat exchanger;

(3) A method utilizing an electric arc.

The contamination of fluid flow is unavoidable in the third method because of the melting electrodes, and the maximum operating temperature is too low in the first method. Therefore, the second method is to be utilized for the low temperature wind tunnel currently being planned. In this method, pebbles of material such as alumina are placed into a cylindrical container and heated by forcing hot air through it. The air to be used in the wind tunnel is pre-heated by exposing it to these pebbles. For the temperature characteristics of this air heater, an analysis of its pre-heating phase has been made in the past (Ref. 1). For the most important heat transfer characteristics during the air heating phase, however, no detailed studies have been made. This report attempts to make a detailed analysis of the thermal characteristics of this type of air heater.

2. Notation /2

A	Non-dimensional constant	
B	Non-dimensional constant	
c	Non-dimensional constant	
C	Heat capacity of a unit volume of pebbles in the heat exchanger	$kcal/m^{3o}C$
c_p	Constant pressure specific heat of the fluid	$kcal/kg^{o}C$

$f=(t_0-t_i)/(T_h-t_i)$		
$G=\rho V$	Flow quantity	kg/m^2 sec
h	Heat conductivity between pebbles and fluid	kcal/sec $m^2 {}^oC$
I_o	Oth degree Bessel function of the first kind	
$K=(T_i-t_i)/(T_h-t_i)$	Non-dimensional constant	
ℓ	Length of heater	m
$\mathcal{L}(\ )$	Laplace transform of ( )	
p	Variable of Laplace transform	
s	Heat conductivity of pebbles of unit volume	$m^2/m^3=m^{-1}$
S	Cross sectional area of heat exchanger	m^2
s_o	Area of heat loss surface per unit volume of the heat exchanger	$m^2 m^3=m^{-1}$
T	Fluid temperature	${}^oC$
T_h	Entrance temperature of heating fluid	${}^oC$
T_i	Entrance temperature of cooling fluid	${}^oC$
t	Temperature of pebbles	${}^oC$
t_o	Initial temperature of pebbles	${}^oC$
t_s	Environmental temperature	${}^oC$
U_o	Heat conductivity of environment	kcal/sec $m^2 {}^oC$
V	Fluid flow velocity	m/sec
x	Distance along the axis of the heat exchanger in the direction of the flow	m

$\alpha = s_0 U_0/sh$ Non-dimensional heat conductivity ratio /3

$\beta = \alpha + 1$

$\gamma = \alpha/\beta$

$\gamma_1 = (\alpha - c)/(\beta - c)$

$\delta = (t - t_s)/(T_h - t_s)$

$\bar{\delta} = \mathcal{L}(\delta)$

$\eta = sh\tau/C$ Non-dimensional time

$\theta = (T - t_s)/(T_h - t_s)$

$\bar{\theta} = \mathcal{L}(\theta)$

$\xi = shx/Gc_p$ Non-dimensional distance

$\xi_1 = shl/Gc_p$ Non-dimensional length of heater

ρ Density of fluid kg/m^3

τ Time sec

$\psi = c_p\rho/C$ Non-dimensional heat capacity ratio

$\Psi = e^{-\xi-\eta} I_0(2\sqrt{\xi\eta})$

3. Basic Equations

The case in which the pebbles are filled evenly in a cylinder of constant cross sectional area is theoretically considered, assuming the following conditions:

(1) Temperature is affected only by time, the flow direction, and distance;

(2) Heat conductivity of the fluid and pebbles are negligible;

(3) The fluid flow velocity is even and constant;

(4) The edge effect is negligible;

(5) The heat conductivities h and U_o are constant;

(6) Heat transfer by radiation is negligible;

(7) Material property constants are independent of the temperature.

/4

The heat lost from the fluid is gained by the pebbles and the environment. Therefore, the following equation can be written (Refer to Fig. 1):

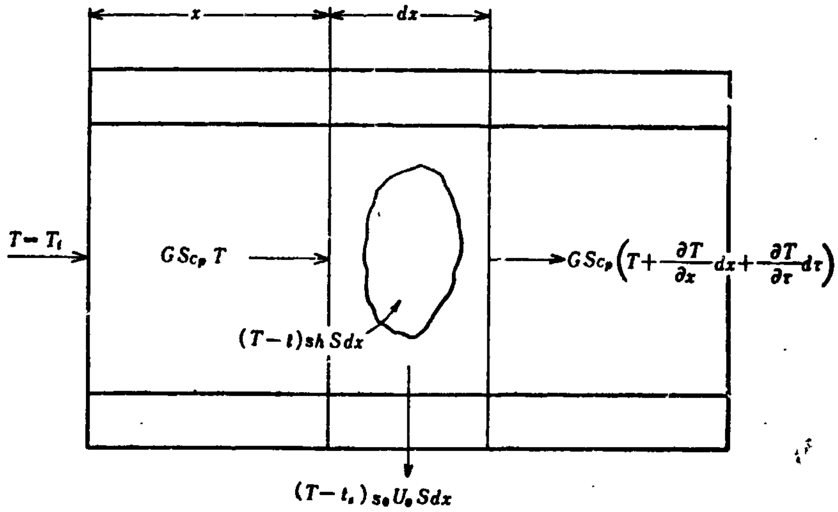


Fig. 1

$$-GSc_p\left(\frac{\partial T}{\partial x}dx+\frac{\partial T}{\partial \tau}d\tau\right)=\{(T-t)sh+(T-t_s)s_0U_0\}Sdx. \tag{1}$$

The relationship between the temperature increase of the pebbles and the heat gained by the pebbles can be expressed as follows (Ref. 1):

$$C\frac{\partial t}{\partial \tau}Sdx=(T-t)shSdx \tag{2}$$

Using the non-dimensional variables, we have

$$\xi=\frac{shx}{Gc_p}, \quad \eta=\frac{sh\tau}{C}, \quad \theta=\frac{T-t_s}{T_h-t_s}, \quad \delta=\frac{t-t_s}{T_h-t_s},$$

and the non-dimensional constants,

$$\psi=\frac{\rho c_p}{C}, \quad \alpha=\frac{s_0U_0}{sh}, \quad \beta=\alpha+1$$

and if the relationship

$$d\tau=\frac{dx}{V}=\frac{\rho dx}{G}$$

is used, equation (1) becomes

$$\frac{\partial\theta}{\partial\xi}+\psi\frac{\partial\theta}{\partial\eta}=\delta-\beta\theta, \tag{3}$$

and equation (2) becomes

$$\frac{\partial \vartheta}{\partial \eta}=\theta-\vartheta \tag{4}$$

4. <u>Heating and Cooling Characteristics at Uniform Temperatures</u>

The case in which the initial temperature of the pebbles and that of the air are uniform and equal ($t_o=T_o$) is considered in this section. In this case, if it is assumed that

$$A=\frac{t_0-t_s}{T_h-t_s}=\frac{T_0-t_s}{T_h-t_s}$$

the initial conditions can be given by the following,

$$\vartheta(\xi,0)=A, \qquad \theta(\xi,0)=A \tag{5}$$

And if it is assumed that

$$K=\frac{T_i-t_s}{T_h-t_s}$$

the boundary conditions can be given as follows: /5

$$\theta(0,\eta)=K \tag{6}$$

where K = 1 when heating the pebbles, and $T_i=t_s$ when cooling the pebbles. Therefore, K=0.

Applying the Laplace transform to equation (3), and substituting the relationship shown in equation (5), the following can be obtained

$$\frac{d\bar{\theta}}{d\xi}+\psi(p\bar{\theta}-A)=\bar{\vartheta}-\beta\bar{\theta} \tag{7}$$

where, $\bar{\theta}=\mathcal{L}(\theta)$, $\bar{\vartheta}=\mathcal{L}(\vartheta)$. Also, from equations (4) and (5), the following can be obtained

$$\bar{\vartheta}=\frac{\bar{\theta}}{p+1}+\frac{A}{p+1} \tag{8}$$

and from equation (6), the following can be obtained

$$\bar{\theta}(0)=\frac{K}{p} \tag{9}$$

Substituting equation (8) into equation (7), we obtain the following :

$$\frac{d\bar{\theta}}{d\xi}+\left(\beta+\psi p-\frac{1}{p+1}\right)\bar{\theta}=\frac{A}{p+1}+\psi A. \tag{10}$$

The solution to equation (10) that satisfies equation (9) is as follows

$$\bar{\theta}=\frac{K}{p}\exp\left\{-\left(\beta+\psi p-\frac{1}{p+1}\right)\xi\right\} \tag{11}$$
$$+\frac{A(1+\psi+\psi p)}{\psi p^2+(\beta+\psi)p+\alpha}\left[1-\exp\left\{-\left(\beta+\psi p-\frac{1}{p+1}\right)\xi\right\}\right]$$

From equation (8), the following can be obtained

$$\bar{\delta}=A\frac{\psi p+\beta+\psi}{\psi p^2+(\beta+\psi)p+\alpha}$$
$$+\left\{\frac{K}{p(p+1)}+\frac{A}{p+1}-A\frac{\psi p+\beta+\psi}{\psi p^2+(\beta+\psi)p+\alpha}\right\}\exp\left\{-\left(\beta+\psi p-\frac{1}{p+1}\right)\xi\right\}$$

In this section, the case in which $\psi>0$ will be considered. (The case in which $\psi=0$ is considered is in the next section.)

Since,

$$D^2=1+\frac{4-2\beta}{\psi}+\frac{\beta^2}{\psi^2}=\frac{4}{\psi}+\left(1-\frac{\beta}{\psi}\right)^2>0$$

if it is assumed that

$$m=\frac{1}{2}\left(-1-\frac{\beta}{\psi}+D\right),\quad n=-\frac{1}{2}\left(1+\frac{\beta}{\psi}+D\right),\quad q=\frac{(\beta/\psi)+1}{D}$$

then,

$$\frac{\psi p+\beta+\psi}{\psi p^2+(\beta+\psi)p+\alpha}=\frac{(\beta/\psi)+1+p}{(p-m)(p-n)}=\frac{1}{2}\frac{1+q}{p-m}+\frac{1}{2}\frac{1-q}{p-n}.$$

Therefore,

$$\bar{\delta}=\frac{A}{2}\left(\frac{1+q}{p-m}+\frac{1-q}{p-n}\right)$$
$$+\left\{\frac{K}{p(p+1)}+\frac{A}{p+1}-\frac{A}{2}\frac{1+q}{p-m}-\frac{A}{2}\frac{1-q}{p-n}\right\}e^{-\beta\xi}\exp\left\{\left(\frac{1}{p+1}-\psi p\right)\xi\right\}.$$

TABLE 1

LAPLACE TRANSFORM TABLE

1 番 号	$F(p)\equiv\mathcal{L}[f(\eta)]$	$f(\eta)$	2 註
1	$\dfrac{1}{p+m}$	$e^{-m\eta}$	文献 2 p. 229 3
2	$\dfrac{1}{p-m}\exp\left\{\left(\dfrac{1}{1+p}-\psi p\right)\xi\right\}$	$\begin{cases} 0 & (0<\eta<\psi\xi) \\ (1+m)e^{m(\eta-\psi\xi)}\int_0^{\eta-\psi\xi} e^{-(1+m)\eta} I_0(2\sqrt{\xi\eta})d\eta & \\ +e^{\psi\xi-\eta} I_0[2\sqrt{\xi(\eta-\psi\xi)}] & (\psi\xi<\eta) \end{cases}$	付 録 4
3	$\dfrac{1}{p(p+1)}\exp\left\{\left(\dfrac{1}{1+p}-\psi p\right)\xi\right\}$	$\begin{cases} 0 & (0<\eta<\psi\xi) \\ \int_0^{\eta-\psi\xi} e^{-\eta} I_0(2\sqrt{\xi\eta})d\eta & (\psi\xi<\eta) \end{cases}$	付 録 4

1 - No.; 2 - Remarks; 3 - Ref. 2, p. 229; 4 - Appendix

From the Laplace transform shown in Table 1, $\Psi=e^{-\xi-\eta}I_0(2\sqrt{\xi\eta})$.

Then,

$$\left.\begin{aligned} \vartheta&=\frac{A}{2}\{(1+q)e^{m\eta}+(1-q)e^{n\eta}\}, && (0<\eta<\psi\xi) \\ \vartheta&=\frac{A}{2}(\{1+q)e^{m\eta}+(1-q)e^{n\eta}\}+Ke^{-\alpha\xi}\int_0^{\eta-\psi\xi}\Psi d\eta && \\ &-\frac{A}{2}(1+q)(1+m)e^{-(1+m\phi)\xi+m\eta}\int_0^{\eta-\phi\xi}e^{-m\eta}\Psi d\eta && \\ &-\frac{A}{2}(1-q)(1+n)e^{-(1+n\phi)\xi+n\eta}\int_0^{\eta-\phi\xi}e^{-n\eta}\Psi d\eta. && (\psi\xi<\eta) \end{aligned}\right\} \tag{12}$$

Similarly, if

$$Q=\left(\frac{2-\beta}{\psi}+1\right)\Big/D$$

then,

$$\bar{\vartheta}=\frac{A}{2}\left(\frac{1+Q}{p-m}+\frac{1-Q}{p-n}\right)+\left(\frac{K}{p}-\frac{A}{2}\frac{1+Q}{p-m}-\frac{A}{2}\frac{1-Q}{p-n}\right)e^{-\beta\xi}\exp\left\{\left(\frac{1}{p+1}-\psi p\right)\xi\right\}$$

Therefore, /7

$$\left.\begin{aligned}
&\theta=\frac{A}{2}\{(1+Q)e^{m\eta}+(1-Q)e^{n\eta}\}, && (0<\eta<\psi\xi)\\
&\theta=\frac{A}{2}\{(1+Q)e^{m\eta}+(1-Q)e^{n\eta}\}\\
&\quad+(K-A)e^{-(\beta-\psi)\xi-\eta}I_0[2\sqrt{\xi(\eta-\psi\xi)}]+Ke^{-\alpha\xi}\int_0^{\eta-\psi\xi}\Psi d\eta\\
&\quad-\frac{A}{2}(1+Q)(1+m)e^{-(\alpha+m\psi)\xi+m\eta}\int_0^{\eta-\psi\xi}e^{-m\eta}\Psi d\eta\\
&\quad-\frac{A}{2}(1-Q)(1+n)e^{-(\alpha+n\psi)\xi+n\eta}\int_0^{\eta-\psi\xi}e^{-n\eta}\Psi d\eta. && (\psi\xi<\eta)
\end{aligned}\right\} \tag{13}$$

When, $\psi << 1$

$$m=-\gamma+\frac{\gamma}{\beta^2}\psi+O(\psi^2), \quad n=-\frac{\beta}{\psi}-\frac{1}{\beta}+O(\psi), \quad q=1+\frac{2\gamma}{\beta}\psi+O(\psi^2)$$

Therefore, if δ, when $\psi=0$, is δ_0, then we have

$$\left.\begin{aligned}
&\delta=\delta_0\left\{1+\frac{\gamma}{\beta}\left(1+\frac{n}{\beta}\right)\psi\right\}, && (0<\eta<\psi\xi)\\
&\delta=\delta_0+\left[A\frac{\gamma}{\beta}e^{-\gamma\eta}\left\{1+\frac{n}{\beta}-e^{-\alpha\xi}\left(\frac{2}{\beta}+\xi+\frac{n}{\beta^2}\right)\right)\int_0^{\eta}e^{\gamma\eta}\Psi d\eta\\
&\quad+e^{-\alpha\xi}\frac{1}{\beta^2}\int_0^{\eta}e^{\gamma\eta}\Psi\eta d\eta\right\}+\xi\Psi e^{-\alpha\xi}\left(\frac{A}{\beta}-K\right)\Bigg]\psi, && (\psi\xi<\eta)
\end{aligned}\right\} \tag{14}$$

where,

$$\begin{aligned}
&\delta_0=Ae^{-\gamma\eta}, && (0<\eta<\psi\xi)\\
&\delta_0=Ae^{-\gamma\eta}+Ke^{-\alpha\xi}\int_0^{\eta}\Psi d\eta-\frac{A}{\beta}e^{-\alpha\xi-\gamma\eta}\int_0^{\eta}e^{\gamma\eta}\Psi d\eta && (\psi\xi<\eta)
\end{aligned}$$

which is a solution already known (Ref. 1).

When the pebbles, which are sufficiently cooled initially (A = 0), are heated (K = 1), the second equation shown in equation (14) becomes as follows:

$$\delta=\delta_0-\psi\xi\Psi e^{-\alpha\xi}=e^{-\alpha\xi}\left(\int_0^{\eta}\Psi d\eta-\psi\xi\Psi\right) \quad (\psi\xi<\eta) \tag{15}$$

5. Heating and Cooling Characteristics of a General Case

The case in which the initial temperature of the pebbles and that of the air are unspecified will be considered in this section. For this condition, assumption $\psi=0$ will be introduced. If it is assumed that,

$$f(\xi)=\frac{t_0-t_i}{T_h-T_i}$$

the initial conditions can be given as follows. /8

$$\vartheta(\xi,0)=f(\xi)$$

The boundary conditions can be given, similarly to the previous section, as follows:

$$\theta(0,\eta)=K$$

Allowing $\psi=0$, and A=f, the following can be obtained from equation (10):

$$\frac{d\bar{\theta}}{d\xi}+\left(\beta-\frac{1}{p+1}\right)\bar{\theta}=\frac{f}{p+1}.$$

The solution to this equation which satisfies equation (9) is as follows,

$$\bar{\theta}=\left[\frac{K}{p}+\frac{1}{p+1}\int_0^\xi f\exp\left\{\left(\beta-\frac{1}{p+1}\right)\xi\right\}d\xi\right]\exp\left\{-\left(\beta-\frac{1}{p+1}\right)\xi\right\}$$

Allowing A=f, the following can be obtained from equation (8),

$$\begin{aligned}\bar{\vartheta}=&\frac{f}{p+1}+\frac{K}{p(p+1)}\exp\left\{-\left(\beta-\frac{1}{p+1}\right)\xi\right\}\\&+\frac{1}{(p+1)^2}\left[\exp\left\{-\left(\beta-\frac{1}{p+1}\right)\xi\right\}\right]\int_0^\xi f\exp\left\{\left(\beta-\frac{1}{p+1}\right)\xi\right\}d\xi\end{aligned}$$

In the subsequent discussion, the case in which the following condition is true will be considered.

$$f=A-Be^{-c\xi} \tag{16}$$

Since,

$$\begin{aligned}\int_0^\xi f\exp\left\{\left(\beta-\frac{1}{p+1}\right)\xi\right\}d\xi=&\frac{A(p+1)}{\beta p+\alpha}\left[\exp\left\{\left(\beta-\frac{1}{p+1}\right)\xi\right\}-1\right]\\&-\frac{B(p+1)}{(\beta-c)p+\alpha-c}\left[\exp\left\{\left(\beta-c-\frac{1}{p+1}\right)\xi\right\}-1\right]\end{aligned}$$

if the condition of $\gamma=\alpha/\beta$, $\gamma_1=(\alpha-c)/(\beta-c)$ is used, the following can be obtained

$$\bar{\vartheta}=\frac{A}{p+\gamma}-\frac{Be^{-c\xi}}{p+\gamma_1}+\frac{Ke^{-\beta\xi}}{p(p+1)}\exp\left(\frac{\xi}{p+1}\right)$$
$$-\frac{A(1-\gamma)e^{-\beta\xi}}{(p+\gamma)(p+1)}\exp\left(\frac{\xi}{p+1}\right)+\frac{B(1-\gamma_1)e^{-\beta\xi}}{(p+\gamma_1)(p+1)}\exp\left(\frac{\xi}{p+1}\right),$$
$$\bar{\theta}=\frac{A}{\beta}\frac{1}{v+\gamma}-\frac{Be^{-c\xi}}{\beta-c}\frac{1}{p+\gamma_1}+Ke^{-\beta\xi}\left\{\frac{1}{p(p+1)}+\frac{1}{p+1}\right\}\exp\left(\frac{\xi}{p+1}\right)$$
$$-\frac{Ae^{-\beta\xi}}{\beta}\left\{\frac{1-\gamma}{(p+\gamma)(p+1)}+\frac{1}{p+1}\right\}\exp\left(\frac{\xi}{p+1}\right)$$
$$+\frac{Be^{-\beta\xi}}{\beta-c}\left\{\frac{1-\gamma_1}{(p+\gamma_1)(p+1)}+\frac{1}{p+1}\right\}\exp\left(\frac{\xi}{p+1}\right)$$

Converting it back and using relationship, $\Psi=e^{-\xi-\eta}I_0(2\sqrt{\xi\eta})$, the following can be obtained. /9

$$\vartheta=Ae^{-\gamma\eta}-Be^{-c\xi-\gamma_1\eta}+Ke^{-\alpha\xi}\int_0^\eta\Psi d\eta-A(1-\gamma)e^{-\alpha\xi-\gamma\eta}\int_0^\eta e^{\gamma\eta}\Psi d\eta$$
$$+B(1-\gamma_1)e^{-\alpha\xi-\gamma_1\eta}\int_0^\eta e^{\gamma_1\eta}\Psi d\eta, \tag{17}$$
$$\theta=A(1-\gamma)e^{-\gamma\eta}-B(1-\gamma_1)e^{-c\xi-\gamma_1\eta}+\{K-A(1-\gamma)+B(1-\gamma_1)\}e^{-\alpha\xi}\Psi$$
$$+Ke^{-\alpha\xi}\int_0^\eta\Psi d\eta-A(1-\gamma)^2e^{-\alpha\xi-\gamma\eta}\int_0^\eta e^{\gamma\eta}\Psi d\eta+B(1-\gamma_1)^2e^{-\alpha\xi-\gamma_1\eta}\int_0^\eta e^{\gamma_1\eta}\Psi d\eta \tag{18}$$

6. Cooling at Even Temperatures

In the case where sufficient heating is performed over a prolonged period of time, and where there is no heat loss from the walls, it becomes, $t_o=T_h$. Therefore, f=A=1 and B=0. If the entrance temperature of the cooling flow is the same as that of the surrounding environment K = 0, equations (17) and (18) become as follows, respectively:

$$\vartheta=e^{-\gamma\eta}-(1-\gamma)e^{-\alpha\xi-\gamma\eta}\int_0^\eta e^{\gamma\eta}\Psi d\eta, \tag{19}$$

$$\theta=(1-\gamma)\left\{e^{-\gamma\eta}-e^{-\alpha\xi}\Psi\cdot\ (1-\gamma)e^{-\alpha\xi-\gamma\eta}\int_0^\eta e^{\gamma\eta}\Psi d\eta\right\} \tag{20}$$

This result agrees with the solution already known (Ref. 1).

7. <u>Cooling at Even Temperatures After Prolonged Heating</u>

Considering the case where the temperature is even and equal to that of the environment, $t_o = t_s$, the following, $f=A=B=0$, is true. Because, $K = 1$, the following equations can be obtained:

$$\delta = e^{-\alpha\xi}\int_0^{\eta} \Psi d\eta, \tag{21}$$

$$\theta = e^{-\alpha\xi}\left(\Psi + \int_0^{\eta} \Psi d\eta\right) \tag{22}$$

These results also agree with the solutions already known (Ref. 1). Now, because of the following relationship,

$$\int_0^{\infty} \Psi d\eta = e^{-\xi}\int_0^{\infty} e^{-\eta} I_0(2\sqrt{\xi\eta})d\eta = 1,$$

$$\lim_{\eta\to\infty} e^{-\eta} I_0(2\sqrt{\xi\eta}) = \lim_{\tau\to\infty} e^{-\tau^2/4\xi} I_0(\tau) = 0$$

after sufficient prolonged heating, the following condition is true (Ref. 1)

$$\theta = \delta = e^{-\alpha\xi} \tag{23}$$

The case of forced air cooling beginning with this condition is to be considered. <u>/10</u>

(1) Heating flow and cooling flow are flowing in the same direction. In this case, the following relationship can be established:

$$f = \delta = e^{-\alpha\xi}$$

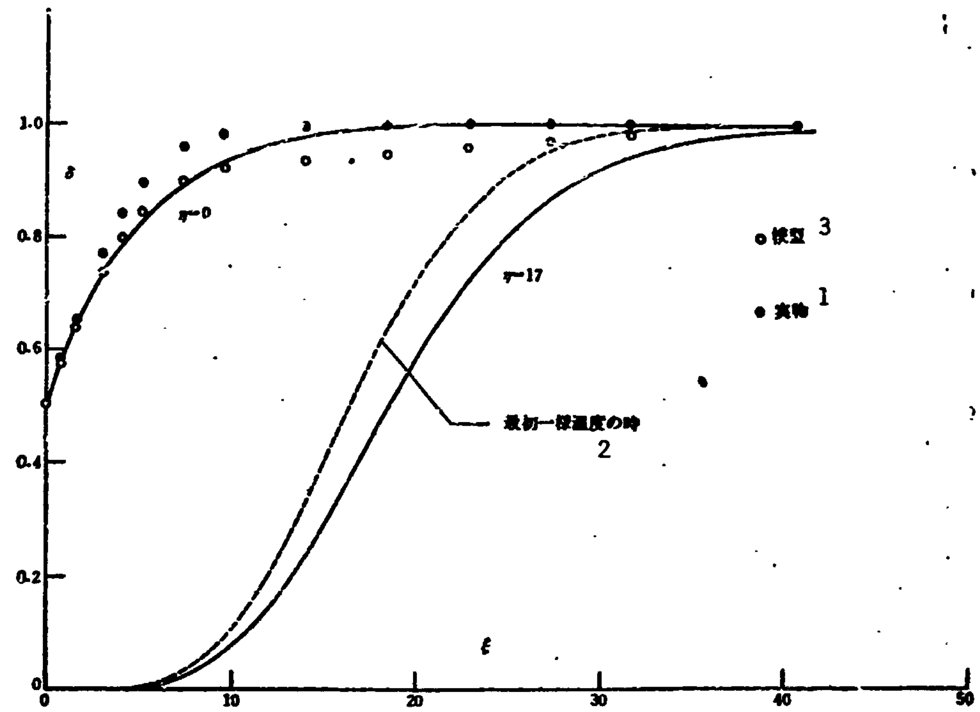
Therefore, $A=0$, $B=-1$, $c=\alpha$, $\gamma_1=0$. Thus, the following results

$$\delta = e^{-\alpha\xi} - (1-K)e^{-\alpha\xi}\int_0^{\eta} \Psi d\eta, \tag{24}$$

$$\theta = e^{-\alpha\xi} + (K-1)e^{-\alpha\xi}\left(\Psi + \int_0^{\eta} \Psi d\eta\right). \tag{25}$$

(2) Heating flow and the cooling flow are flowing in the opposite direction.

Using the length of the heater, ℓ, the following equation, $\xi_1 = shl/Gc_p$, can be obtained. Then, because the following relationship holds



/11

Figure 2

Cooling Characteristics (Pebble temperature)

(1) - Actual heater; (2) - The case of uniform initial temperature; (3) - Model.

$$j = e^{-\alpha\xi_1}e^{\alpha\xi}$$

the following can be obtained:

$$A=0,\ B=-e^{-\alpha\xi_1},\ c=-\alpha,\ \gamma_1=2\alpha/(2\alpha+1)$$

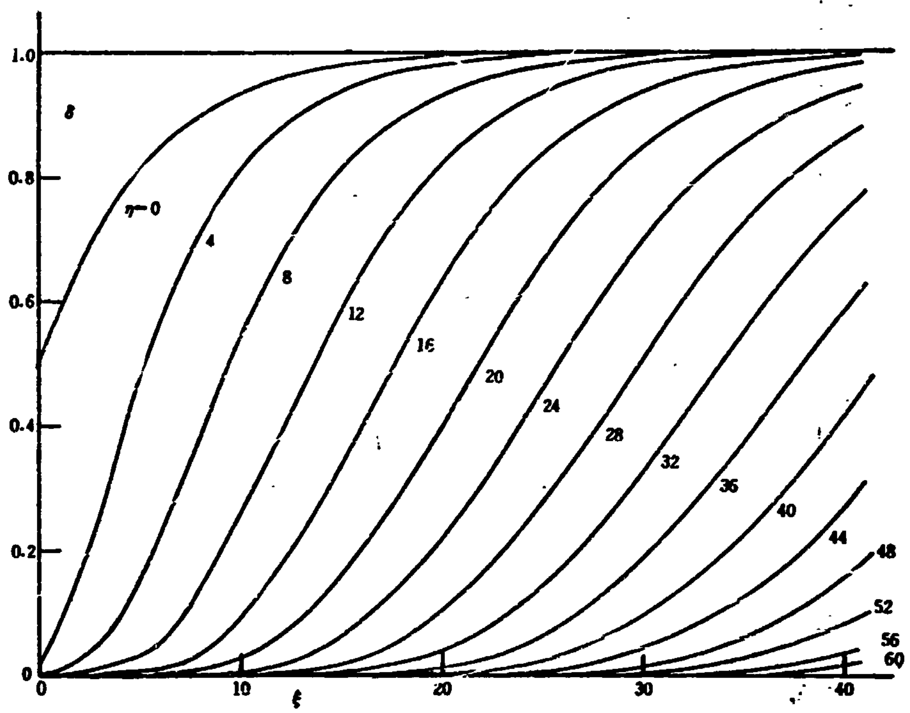
Thus,

$$\delta=e^{-\alpha\xi_1}e^{\alpha\xi-\gamma_1\eta}+Ke^{-\alpha\xi}\int_0^\eta\Psi d\eta-(1-\gamma_1)e^{-\alpha\xi_1}e^{-\alpha\xi-\gamma_1\eta}\int_0^\eta e^{\gamma_1\eta}\Psi d\eta, \tag{26}$$

$$\begin{aligned}\theta=&(1-\gamma_1)e^{-\alpha\xi_1}e^{\alpha\xi-\gamma_1\eta}+\{K-(1-\gamma_1)e^{-\alpha\xi_1}\}e^{-\alpha\xi}\Psi\\&+Ke^{-\alpha\xi}\int_0^\eta\Psi d\eta-(1-\gamma_1)^2e^{-\alpha\xi_1}e^{-\alpha\xi-\gamma_1\eta}\int_0^\eta e^{\gamma_1\eta}\Psi d\eta.\end{aligned} \tag{27}$$

8. Calculation Examples

As an actual calculation example, a calculation was made for the heater to be used with the super-sonic wind tunnel currently under construction,



/12

Figure 3

Cooling Characteristics (Pebble temperature)

and for this heater's preliminary experimental mockup. The constants for this case are shown in Table 2. For these values of the constants, the temperature distribution of the pebbles was calculated using equation (15) for the case in which it was heated until the value of delta reached 1/2 at the end of heating. The results obtained are shown in Fig. 2 as o (for the model) and • (for the actual heater). Using pebbles having this type of temperature distribution, we investigated the cooling characteristics for the case in which cooling air flowed in the opposite direction to the heating air flow. The temperature distribution was approximated by equation (16) assuming the following values:

$$A=1.0001260, \quad B=0.50012594, \quad c=0.20489647$$

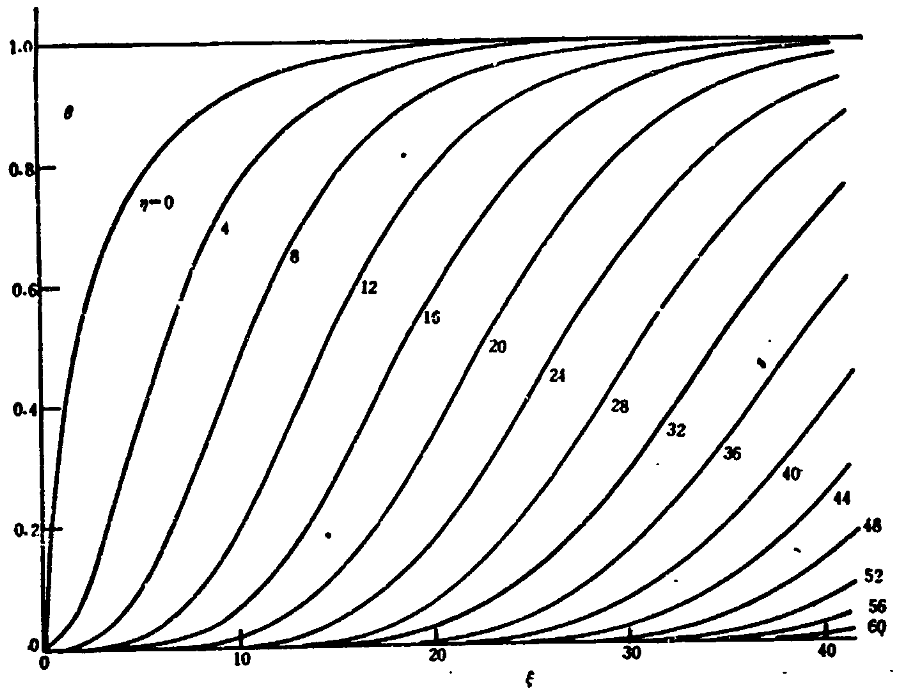


Figure 4

Cooling Characteristics (Air temperature)

This result is shown in the same figure as solid line (η=Q). Using this result and setting γ=K=0, the results obtained from equation (17) are shown in the same figure as the solid line (η=17).* For purposes of reference, the results of η=17 obtained from equation (19) assuming an even initial temperature, δ=1, are shown by the dotted line. These results show that there is a considerable difference in the temperature distribution. Figure 3 shows the temperature distributions of the pebbles where η=0(4)60. The air temperature, θ , is shown in Fig. 4. In Fig. 5, the temperature of air flowing out of the end of the cooler is shown as a function of the non-dimensional time, η . The solid line shows the results obtained from equation (20).

---

*The α during cooling is small as shown in Table 2, and the results differ very little even if γ=0.

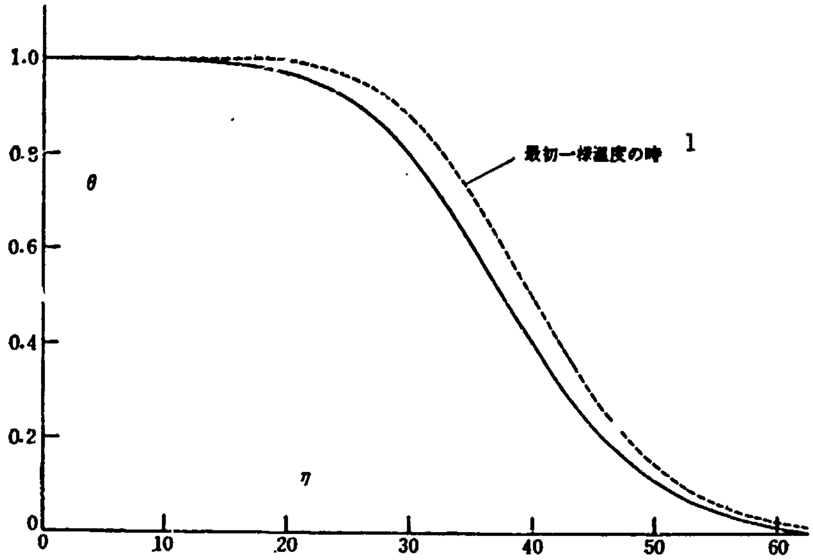


Figure 5

Exit Flow Temperature
(1) - The case of uniform initial temperature

In closing, I wish to thank Professor Isao Imai of the University of Tokyo, Aero-navigation Laboratory and Dr. Tomio Yamaguchi of the Shinmitsubishi Heavy Industry Co., Kobe Ship-yard Laboratory who gave valuable advice during this research.

TABLE 2

	$\alpha \times 10^4$	ξ/x (m^{-1})	η/t	ϕ
Heating	model 5.88 actual 1.02	92.1	141 (h^{-1})	0.003
Cooling	model 1.71 actual 0.297	20.22	8.09 (min^{-1})	0

(Basis of calculation of results shown in Table 2)

(1) s_0 If the diameter of the heat exchanger is D(m), the surface area per unit length is πD, and the volume is $\pi D^2/4$. Therefore,

$$s_0 = \frac{\pi D}{\frac{\pi}{4}D^2} = \frac{4}{D}.$$

(2) s If the diameter of the pebble is d(m), and the gap factor of the filling layer is e, then,

$$s = \frac{6(1-e)}{d}.$$

For the most densely packed case, e is $1-(\pi/2\sqrt{6})=0.3587$.

(3) c_p 0.265 kcal/kg^{o}C.

(4) C If the apparent specific gravity of the alumina is assumed to be 3.3, and the specific heat is assumed to be 0.244 kcal/kg^{o}C, then,

$$C = 3300 \times 0.6413 \times 0.244 = 516.4 \text{ (kcal/m}^3\,{}^\circ\text{C)}.$$

(5) G, h Heating $G = 2982$ kcal/m^2 h, $h = 180$ kcal/m^2 h °C,

Cooling $G = 46768$ kg/m^{2}h, $h = 620$ kcal/m^2 h °C.

(6) U_o, D, d

	U_0 (kcal/m^2 h °C)	D (m)	d (m)
model	3.21	0.3	9.5×10^{-3}
actual	2.60	1.4	9.5×10^{-3}

Appendix /14

The second and third Laplace transforms shown in Table 1 will be derived. The table of Laplace transforms used as the basis, extracted from (Ref. 2) are shown in Table 3.

(i) Derivation of second transforms

$$\frac{1}{p-m}\exp\left\{\left(\frac{1}{p+1}-\psi p\right)\xi\right\} = \frac{1}{p-m}\left\{\exp\left(\frac{\xi}{p+1}\right)-1\right\}e^{-\psi\xi p} + \frac{1}{p-m}e^{-\psi\xi p}. \quad \text{(A.1)}$$

Now, from 1 and 4 of Table 3, we have

$$\exp\left(\frac{\xi}{p+1}\right)-1 = \mathcal{L}\left[e^{-\eta}\left(\frac{\xi}{\eta}\right)^{1/2} I_1(2\sqrt{\xi\eta})\right]. \quad \text{(A.2)}$$

TABLE 3

LIST OF LAPLACE TRANSFORMS

No.	$F(p)\equiv\mathcal{L}[f(\eta)]$	$f(\eta)$	Pg. no. of Ref.2
1	$F(p-A)$	$e^{A\eta}f(\eta)$	p. 129
2	$F_1(p)F_2(p)$	$\int_0^\eta f_1(z)f_2(\eta-z)dz$	p. 131
3	$\frac{1}{p-m}e^{-Ap}\quad(A>0)$	$\begin{cases}0 & (0<\eta<A)\\ e^{m(\eta-A)} & (A<\eta)\end{cases}$	p. 242
4	$\exp\left(\frac{\xi}{p}\right)-1$	$\left(\frac{\xi}{\eta}\right)^{1/2}I_1(2\sqrt{\xi\eta})$	p. 244
5	$\frac{1}{p}\exp\left(\frac{\xi}{p}\right)$	$I_0(2\sqrt{\xi\eta})$	p. 245

and from 3 of Table 3,

$$\frac{1}{p-m}e^{-\psi\xi p}=\begin{cases}\mathcal{L}[0], & (0<\eta<\psi\xi)\\ \mathcal{L}[e^{m(\eta-\psi\xi)}]. & (\psi\xi<\eta)\end{cases} \tag{A.3}$$

From these two equations and from 2 of Table 3, we have

$$\left\{\exp\left(\frac{\xi}{p+1}\right)-1\right\}\frac{1}{p-m}e^{-\psi\xi p}=\mathcal{L}\left[\int_0^\eta e^{-z}\left(\frac{\xi}{z}\right)^{1/2}I_1(2\sqrt{\xi z})\Delta(\eta-z)\,dz\right],$$

where

$$\Delta(\eta-z)=\begin{cases}0, & (\eta-\psi\xi<z)\\ e^{m(\eta-z-\psi\xi)}. & (0<z<\eta-\psi\xi)\end{cases}$$

Therefore,

$$\frac{1}{p-m}\left\{\exp\left(\frac{\xi}{p+1}\right)-1\right\}e^{-\psi\xi p}=\begin{cases}\mathcal{L}[0], & (0<\eta<\psi\xi)\\ \mathcal{L}\left[e^{m(\eta-\psi\xi)}\int_0^{\eta-\psi\xi}e^{-(1+m)\eta}\left(\frac{\xi}{\eta}\right)^{1/2}I_1(2\sqrt{\xi\eta})d\eta\right]. & (\psi\xi<\eta)\end{cases}$$

Substituting this equation and equation (A.3) into equation (A.1),

/15

$$\frac{1}{p-m}\exp\left\{\left(\frac{1}{p+1}-\psi p\right)\xi\right\}$$
$$=\begin{cases}\mathcal{L}[0], & (0<\eta<\psi\xi)\\ \mathcal{L}\left[e^{m(\eta-\psi\xi)}\left\{1+\int_0^{\eta-\psi\xi}e^{-(1+m)\eta}\left(\frac{\xi}{\eta}\right)^{1/2}I_1(2\sqrt{\xi\eta})d\eta\right\}\right]. & (\psi\xi<\eta)\end{cases} \tag{A.4}$$

Now,

$$\int_0^{\eta-\psi\xi}e^{-(1+m)\eta}\left(\frac{\xi}{\eta}\right)^{1/2}I_1(2\sqrt{\xi\eta})d\eta=\int_0^{\eta-\psi\xi}e^{-(1+m)\eta}\frac{dI_0}{d\eta}d\eta$$
$$=e^{(1+m)(\psi\xi-\eta)}I_0[2\sqrt{\xi(\eta-\psi\xi)}]-1+(1+m)\int_0^{\eta-\psi\xi}e^{-(1+m)\eta}I_0(2\sqrt{\xi\eta})d\eta.$$

Therefore, the second transform in Table 1 is derived.

(ii) <u>Derivation of third transform</u>

From 1 and 5 of Table 3, we have

$$\frac{1}{p+1}\exp\left(\frac{\xi}{p+1}\right)=e^{-\eta}I_0(2\sqrt{\xi\eta})\,.$$

Therefore, using 2 of Table 3,

$$\frac{1}{p(p+1)}\exp\left\{\left(\frac{1}{p+1}-\psi p\right)\xi\right\}=\frac{1}{p+1}\exp\left(\frac{\xi}{p+1}\right)\frac{1}{p}e^{-\psi\xi p}$$
$$=\mathcal{L}\left[\int_0^{\eta}e^{-z}I_0(2\sqrt{\xi z})\Delta(\eta-z)dz\right],$$

where,

$$\Delta(\eta-z)=\begin{cases}0, & (\eta-\psi\xi<z)\\ 1. & (0<z<\eta-\psi\xi)\end{cases}$$

Therefore, the third transform in Table 3 can be derived.

REFERENCES

1. Dabora, E. K. Regenerative Heat Exchangers with Heat-Loss Consideration, ASME Paper No. 58-SA-29, 1958.

2. Staff of the Bateman Manuscript Project. Tables of Integral Transforms, Vol. 1, McGraw-Hill Co. New York, 1954.

3. Aluminum Co. of America. Hydrated, Calcined, Tabular Aluminas & Calcium Aluminate Cement, 1961. ALCOA Chemicals Product Data GA2A.

/16

NAL TR-52	I Nisiki Hayashi
National Aerospace Laboratory	II NAL TR-52
Thermal Characteristics of a Pebble-Bed Heater	III 536.2
October 1963 15 pages	621.43

The transient thermal characteristics of a heater, having pebbles evenly filled in a cylinder of constant cross sectional area, in which a fluid of different temperature flowed, were calculated assuming the following conditions. Temperature is a function of time and distance in the direction of flow only; the flow velocity is uniform and constant; the heat transfer due to heat conduction and radiation, and the edge effect are negligible; the heat conductivity is constant; and the material property constants are independent of temperature.

A detailed analysis was made for the case in which the initial temperatures of the pebbles and the fluid were equal and uniform. Also, a solution was derived for the case in which the heat capacity of the fluid is negligible compared with that of the pebbles, and the initial temperature distribution of the pebbles can be expressed as $A\text{-}Be^{-c§}$ (where A, B, c are constants, and § is non-dimensional axial distance).*

---

*Note: There are four identical parts of what appears to be a library index card on this page.

List of Reports Previously Published

TR-20	A Numerical Method for Solving Blasius' Type Differential Equation	January, 1962, Kazuo Higuchi, Hayato Togawa
TR-21	An Investigation of Two-dimensional Control Surface at Transonic Speed (I)	January, 1962, Nobuhiko Kamiya
TR-22	On the Free Lateral Vibration of a Beam with Variable Cross Section	January, 1962, Tadahiko Kawai, Hayato Togawa, Yoichi Hayashi
TR-23	Monte Carlo Solutions of the Boundary Value Problems for Some Types of Partial Differential Equations	February, 1962, Kazuo Higuchi, Toshiyuki Takahashi, Ryoso Torimi
TR-24	Test Facilities of Turbo-jet Engine Components at N. A. L.	February, 1962, National Aerospace Laboratory, Prime Mover Department
TR-25	Design and Construction of the National Transonic Wind Tunnel	March, 1962, National Aerospace Laboratory
TR-26	Preliminary Tests of Supersonic Blowdown Wind Tunnel	March, 1962, Toshio Kawasaki, Yujiro Okabe, Yoshikazu Ogata, Hisashi Ando
TR-27	An Approximate Analysis for Unsteady Transonic Flow	July, 1962, Iwao Hosokawa, Ko Miyoshi
TR-9T	Studies on the Small Disturbance Theory of Transonic Flow (I) --Nonlinear Correction Theory--	July, 1962, Iwao Hosokawa
TR-28	On Fatigue Cracks	August, 1962, Yukio Takenaka
TR-29	On the Design and Construction of the 1m x 1m Supersonic Blow-down Wind Tunnel	October, 1962, Aerodynamics Section II

TR-30	On the Natural Vibration of Plate-Like Wings	October, 1962, Tadahiko Kawai, Taketoshi Hanawa, Hayato Togawa, Toshiyuki Takahashi, Shinichi Koshiide
TR-31	On the Instability and Small Natural Torsional Vibration of a Thin Wing under a Thermal Stress	November, 1962, Tadahiko Kawai, Yoichi Hayashi, Hayato Togawa
TR-32	A Method of Analysis on the Compressive Strength of Stiffened Plates	November, 1962, Tadahiko Kawai, Koichi Egawa
TR-33	Reported Load Testing Rigs for Full Scale Aircraft Wing Structures	December, 1962, Kazuyuki Takeuchi Soshiro Iida, Koichi Ono
TR-34	An Intestigation of High Speed Axial Flow Compressor (I) --The Selection of Compressor Cascade--	January, 1963, Masakatsu Matsumoto, Koichi Ohyama, Toshio Miyaji
TR-35	An Investigation of High Speed Axial Flow Compressor (II) --Design and Over-all Performance of a Single Stage Axial Flow Compressor--	January, 1963, Masakatsu Matsumoto, Toshio Miyaji, Koichi Ohyama, Noboru Yoshida, Hideo Nishiwaki, Keiso Iwabe
TR-36	Studies of Surface Heat Transfer Using a Hypersonic Shock Tunnel	January, 1963, Ksamu Wada, Toshiichi Matsuzaki
TR-37T	Studies of the Flow in a Low Pressure Hypersonic Shock Tunnel Using an Electron-Beam Densitometer	January, 1963, Isamu Wada
TR-38	Strength of Rotating Discs of Brittle Material like Cast Iron	February, 1963, Kazuo Sato, Fumio Nagai
TR-39	A Study of High Intensity Combustor (I) --Its Qualitative Analysis--	February, 1963, Sadakichi Ohtsuka, Kunio Suzuki

TR-40	Repeated Load Testing Facility for Full-Scale Aircraft Fuselage Structures	February, 1963, Kazuyuki Takeuchi, Ikuo Kawashima, Toshio Nohara
TR-41	Calibration of Radiometer	February, 1963, Yukihiko Takenaka, Koichi Egawa, Koichi Ogawa
TR-42	A Similar Solution of Unsteady Laminar Boundary Layer and Its Stability Characteristics	July, 1963, Yasujiro Kohashi, Ei Onji
TR-43	On the Effectiveness of Control Surfaces in Supersonic Flow	February, 1963, Toshi Kawasaki
TR-44	Some Notes about the Effect of Tunnel Configuration and Testing Technique on Compressor Cascade Performance	February, 1963, Hiroshi Kondo, Mitsuhiro Minota, Hajime Sakaguchi, Toshio Yamazaki
TR-45	A Numerical Method for the Eigenvalue Problem $(\Sigma\lambda^k A_k)\ x = 0$	April, 1963, Hayato Togawa
TR-46	On the Vibration Analysis of Aircraft Wings	June, 1963, Tadahiko Kawai, Hideo Izumi, Hayato Togawa, Yoichi Hayashi
TR-47	Influence of Cariolis' Force on the Burst of Rotating Disc of Cart Iron	July 1963, Kazuo Sato, Fumio Nagai
TR-48	Effects of Surface Curvature on Laminar Boundary-Layer Flow	August, 1963, Nisiki Hayashi
TR-49	An Investigation of High Speed Axial Flow Compressor	September, 1963, Masakatsu Matsumoto, Toshio Miyaji, Koichi Ohyama, Noboru Yoshida, Hideo Nishiwaki, Keiso Iwabe
TR-50	Numerical Method for Boundary Value Problems of Partial Differential Equations by Boundary Contraction	September, 1963, Kazuo Higuchi, Tsutomu Nomi
TR-51	Human Control Ability of the Statically Unstable System	September, 1963, Shun Takeda

NASA TT F-10,188

NATIONAL AEROSPACE LABORATORY REPORT NO. 52

October 1963

Publisher: National Aerospace Laboratory
1,380 Shindaiji-cho, Chofu-shi, Tokyo
Telephone: Musashino (0422)(3)5171 (Representative)

Printer: Kasai Publication Printing Co.
1-53, Shiba Minami-Sakuma-cho, Minato-ku, Tokyo

Scientific Translation Service
4849 Tocaloma Lane
La Canada, California